LAGUARDIA COMMUNITY COLLEGE
CITY UNIVERSITY OF NEW YORK
Department of Mathematics, Engineering, and Computer Science

MAT 221 – INTRODUCTION TO PROBABILITY

4 Hours, 4 Credits

Pre-Requisite: MAT 203 or Waiver

Catalog Description:

This course is an introduction to the theory of probability. The topics studied are basic theorems of probability, permutations and combinations, discrete and continuous random variables, univariate probability distributions, multivariate probability distributions, jointly distributed random variables, sequences of independent identically distributed random variables, method of moments, the moment-generating function, the central limit theorem, the law of large numbers, confidence intervals, hypothesis testing, chi-square methods, and simple linear regression.

Purposes and Goals:

Upon the completion of this course, students should be able to:

1. Apply the concepts of probability theory to real-life situations to evaluate risks.
2. Be able to model various real-life processes using probability distributions.
3. Transfer to four-year colleges and universities and pursue upper division courses and academic programs such as, actuary sciences, management science, and engineering program.
4. Take the first exam of the Society of Actuary and the Casualty Actuarial Society.

Instructional Objectives:

The instructor is expected to:

1. Introduce the basic elements of probability, and explain the use of permutations and combinations in computing probabilities of various events.
2. Discuss mutually exclusive events, independent events, and Baye's theorem. Explain how to compute marginal and conditional probabilities, and discuss their application in a risk management context.
3. Introduce discrete and continuous random variables. Demonstrate how to compute the expected value of a random variable and the expected value of a function of a random variable.
4. Familiarize students with discrete and continuous probability distributions such as: binomial, negative binomial, geometric, hypergeometric, Poisson, uniform, exponential, chi-square, beta, lognormal, beta, gamma, Weibull, and normal distributions. Solve application problems involving these distributions.
5. Discuss independent random variables and probability distributions of sums of independent random variables.
6. Introduce the concept of jointly distributed random variables, and explain how to find probability distributions of jointly distributed random variables. Compute conditional probabilities from joint probability distribution functions.
7. Discuss order statistics and its applications.
8. Introduce the concept of moment generating functions, and derive expected value and other moments from a moment generating function.
9. Derive moments for linear combinations of independent random variables and simple transformations of random variables.
10. Introduce multivariate normal random variable and its distribution.
11. Introduce limit theorems (central limit theorem, law of large numbers, Chebyshev's inequality) and discuss their applications.
12. Discuss the concepts of estimation and hypothesis testing. Explain how normal distribution and Student's t distributions can be applied to estimate population parameters and conduct hypothesis tests about parameters. Instructor should clarify

the assumptions and limitations of the statistical techniques that are based on these distributions.

13. Discuss procedures of hypothesis testing using Chi-square distribution.
14. To introduce the students to regression and correlation analysis.

Performance Objectives:

As a result of successful completion of this course, students should be able to:

1. Compute probability of events using permutations and combinations.
2. Distinguish between mutually exclusive events and independent events, apply Baye's theorem to compute conditional probabilities, and solve problems in risk management involving marginal and conditional probabilities.
3. Identify discrete and continuous random variables. Compute expected values of a random variable and of a function of a random variable.
4. Identify various discrete and continuous probability distributions and apply them to solve problems.
5. Compute probability distributions of sums of independent random variables.
6. Derive probability distributions of jointly distributed random variables. Compute conditional probabilities from joint probability distribution functions.
7. Explain order statistics and compute probabilities of order statistics in application problems.
8. Find expected value and other moments from a moment generating function.
9. Compute moments for linear combinations of independent random variables and simple transformations of random variables.
10. Find joint and conditional probability distribution for the multivariate normal random variable.
11. Apply limit theorems in solving application problems.
12. Use the normal or the Student's t-distribution to estimate population parameters and conduct hypothesis tests.
13. Apply the chi-square goodness-of-fit test to make inferences about a distribution of a qualitative variable or discrete quantitative variable that has only finitely many possible values.

14. Determine correlation coefficients, identify and characterize linear correlation, determine the probable form of the relationship between dependent and independent variables, and obtain an equation that can be used for making predictions.

Attendance:

Students are expected to attend all class meetings. Students are responsible for all information, material, and assignments covered in class regardless of class attendance. Students should consult the college catalog to find out the terms and conditions under which a WU, and incomplete, or an F grade may be given by an instructor.

Textbooks:

1. *A First Course in Probability*; Eighth Edition, Sheldon Ross; Pearson Prentice Hall, Copyright 2010 – Pearson Education, Inc.
2. Supplement (chapter 4, 9.1, 9.2, 9.3, 10.1, 10.2, 10.3, 12.1, 12.2): *Fundamentals of Statistics*; Second Edition, Michael Sullivan, III; Pearson Prentice Hall, Copyright 2008 – Pearson Education, Inc.
2. Reference: *Applied Statistics and Probability for Engineers*, 4th Edition, Douglas C. Montgomery and George C. Runger, Wiley 2006.
3. Reference: *Probability for Risk Management* – Matthew J. Hassett, and Donald G. Stewart; ACTEX Publications. ISBN 1-56698-347-9

(*Note: The pages numbers and problem numbers may differ in the newer edition of the books. Please refer to the topics listed in the syllabus*)

Course Grading:

Course grades will be determined in the following manner:

Test #1	15%
Test #2	15%
Test #3	15%
Quizzes, Assignments	10%
Projects	15%
Final Examination	30%

Course Content Outline

LESSON	TOPIC	SECTION	PAGE	SUGGESTED EXERCISES & HOMEWORK
1 – 2	**Chapter 1:** Combinatorial Analysis *The basic principle of counting, Permutations, Combinations, Multinomial coefficient*	1.1 – 1.5	1 – 12	4, 5, 7, 9, 11, 12, 13, 18, 21, 24, 26, 27 HW: 4, 5, 9, 12, 18
3 – 4	**Chapter 2:** Axioms of Probability *Sample space and events, axioms of probability, some simple propositions, sample spaces having equally likely events, Probability as a continuous set function*	2.1 – 2.6	22 – 48	3, 5, 9, 10, 12, 13, 19, 28, 37, 39, 40, 44, 49 HW: 10, 12, 13, 19, 37, 39
5 – 6	**Chapter 3:** Conditional Probability and Independence *Conditional probabilities, Baye's formula, independent events*	3.1 – 3.4	58 – 93	1, 6, 15, 17, 18, 19, 21, 52, 53, 57, 71 HW: 17, 18, 20, 21, 52
7 – 10	**Chapter 4:** Random Variables *Random variables, discrete random variables, expected value, expectation of a function of a random variable, variance, The Bernoulli and binomial random variable, the Poisson random variable, The Geometric random variable, the Negative Binomial random variable, the Hypergeometric random variable, expected value of sums of random variables, properties of cumulative distribution function*	4.1 – 4.10	117 - 170	1, 4, 13, 17, 18, 19, 21, 23, 27, 28, 35, 38, 40, 42, 48, 51, 52, 53, 54, 57, 58, 63, 64, 71, 75 HW: 1, 19, 28, 40, 48, 52, 63, 64
11	**Review for Test # 1**			
12	**Test # 1**			

13 – 15	**Chapter 5:** Continuous Random Variables *Expectation and variance of continuous random variable, the Uniform random variable, Normal random variable, Exponential random variable, Hazard rate function, the Gamma distribution, the Weibull distribution, the Cauchy distribution, the Beta distribution, the distribution function of a random variable*	5.1 – 5.7	186 – 222	1, 2, 4, 5, 7, 8, 10, 13, 15, 16, 19, 20, 24, 25, 31, 32, 34, 35, 36, 39 HW: 4, 8, 24, 32, 36, 39
16 – 20	**Chapter 6:** Jointly Distributed Random Variables *Joint distribution function, independent random ariables, sums of independent random variables (Gamma, Normal, Poisson, Binomial, Geometric), conditional distribution (discrete and continuous cases), order statistics, joint probability distribution of functions of random variables*	6.1 – 6.7	232 – 282	6, 8, 10, 12, 20, 54, 56, 58 HW: 1, 9, 11, 13, 21, 55, 58
21	**Chapter 7:** Properties of Expectation *Expectation of sums of random variables*	7.1, 7.2	297 – 315	1, 5, 6, 10, 16, 30, 36 HW: 4, 33, 38
22	**Review for Test # 2**			
23	**Test # 2**			
24 – 27	**Chapter 7:** Properties of Expectation *Covariance, variance of sums, and correlation, conditional expectation, conditional expectation and prediction, moment generating functions, joint moment generating function*	7.4 – 7.7	331 – 265	48 HW: 50, 51
28 – 30	**Chapter 8:** Limit Theorems *Chebyshev's Inequality and Weak Law of Large Numbers, The Central Limit Theorem, Large Law of Large Numbers*	8.1 – 8.4	388 – 403	4, 5, 9, 10 HW: 6, 7

31 – 33	**Supplement:** Confidence Interval for Single Population Mean	*Sullivan*, Chapters 9.1, 9.2		Instructor handouts
34 – 36	**Supplement:** Hypothesis Testing for Single Population Mean *Type I and Type II Errors, P-value*	Sullivan, Chapters 10.1 – 10.3		Instructor handouts
37 – 39	**Supplement:** Chi-Square Methods *Goodness-of-fit test, Test of Independence, Test of Homogeneity*	*Sullivan*, Chapter 12		Instructor handouts
40 – 41	**Supplement:** Correlation *Draw and interpret scatter diagrams, Linear Correlation Coefficient r, Properties of r, Compute and interpret r, Determine whether there is a linear relation between two variables*	*Sullivan*, Chapter 4		Instructor handouts
42	**Review for Test # 3**			
43	**Test # 3**			
44 – 46	**Supplement:** Linear Regression *Find the least square regression line, y-intercept and Slope, Prediction with regression equation, Interpreting the Regression equation, Residuals*	*Sullivan*, Chapter 4		Instructor handouts
47 - 48	Final Examination Review			
37	**Final Examination**			