

The Yarkovsky Effect

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April 2, 2018

Abstract

The Yarkovsky effect is the result of a force generated due to uneven emission of thermal radiation, which is usually associated with smaller objects in space such as asteroids and meteorites. The Yarkovsky effect has two components: seasonal effect which manifests when the object is not spinning and diurnal effect where it takes effect when the object is rotating. In this work, we calculated the seasonal effect assuming a spherical object was at one astronomical unit distance away from the Sun. The formula was derived for spherical objects of various radii, and it was found that the gravitational force from the Sun was always stronger than the force of the Yarkovsky effect. It was also found that as the radius became smaller, the Yarkovsky effect became more prominent compared to the gravitational force. However, the stronger force of the seasonal effect on extremely small particles (of a radius less than 10^{-6} m) does not correspond to what is actually observed. This can be explained due to not taking into consideration the transfer of heat to the opposite side of the sphere, which will make smaller objects less affected by the Yarkovsky effect.

1 Introduction

The Yarkovsky effect is the effect where uneven emission of thermal radiation causes a force, which results in a deviation from a projected trajectory of a body of object in space (Daniel T. Britt, 2007). All bodies in space at a non-zero temperature give off thermal radiation. The thermal radiations being emitted consists of photons which will have a momentum, generating an opposite force according to Newton's Second Law. The Yarkovsky effect is usually associated with meteoroids or asteroids due to size being a

large factor of its effect (Badescu, 2013). When an object is too large, the Yarkovsky effect is very minimal compared to gravitational forces. On the other hand, when the object is too small, due to the nature of the thermal conductivity, the Yarkovsky effect is also very minimal. It is also heavily associated with meteoroids or asteroids due to the difficulty of prediction of trajectory because of the deviation due to the larger effects of the Yarkovsky effect on them.

There are two components of the Yarkovsky effect, the seasonal effect, and the diurnal effect which is commonly known as the Yarkovsky-O'Keefe-Radzievskii-Paddack (YORP) effect (Badescu, 2013). The two effects are very similar in that the thermal radiation will be generating a force. The seasonal effect is produced when the object is not rotating and larger amounts of the sun's radiation are exposed on one side, and the thermal radiation emitted from the object will push it farther away from the light source. The YORP effect is very similar, but takes into account the rotation of the object. When the object is rotating, a portion of the thermal radiation being emitted from the object will spillover to either the front or the back of the object depending on the rotating direction. Due to this spillover, the object's rotational speed will either accelerate or decelerate.

In this research, we calculate the force of the seasonal effect (here on it is referred to as simply the Yarkovsky force) as a function of radius of a spherical object in space and temperature, which is determined by the distance from the source of thermal radiation.

2 Theory

To derive the force due to the Yarkovsky effect, we first calculate the force exerted on a particle due to radiation of light. If a particle emits N_γ photons with p_γ momentum each, then the particle loses the total momentum $p_\gamma N_\gamma$, so the force on the particle can be written as:

$$F_{rad} = \frac{p_\gamma N_\gamma}{t}, \quad (1)$$

where t is the time of the emission. It is convenient to relate the radiation force (1) to the power of radiation W_{rad} , which is the energy radiated per unit of time

$$W_{rad} = \frac{E_\gamma N_\gamma}{t}, \quad (2)$$

where E_γ is the energy of a single photon. Using the relation between the photon energy and momentum, $p_\gamma = E_\gamma/c$, and Eqs. (1) and (2) we get

$$F_{rad} = \frac{p_\gamma N_\gamma}{t} = \frac{E_\gamma N_\gamma}{ct} = \frac{W_{rad}}{c}, \quad (3)$$

here c is the speed of light in vacuum (3×10^8 m/s).

The power of thermal radiation is well studied and can be described as:

$$W_{rad} = \varepsilon \sigma T^4 A, \quad (4)$$

where A is the surface area, ε is the emissivity, and σ is the Stephan-Boltzman constant ($\sigma = 5.6704 \times 10^{-8}$ W m⁻²K⁻⁴). The emissivity factor describes how well bodies absorb or emit thermal radiation, and for this research we will assume $\varepsilon = 1$, for the sake of simplicity.

Using Eqs.(3-4), the Yarkovsky force will be calculated for a spherical object. The light is assumed to be emitted perpendicular to the surface, so the corresponding force will have x , y , and z components. Assuming that only one side of the particle is illuminated by the sunlight, only half of the sphere will be subject to the thermal radiation source. The resultant force will be directed along z direction, all of the x and y components will be canceled out due to the equal and opposite forces on the opposing side, leaving only the z component of the original force remaining as shown in Fig. 1, and can be calculated as $F_z = F_0 \cos \theta$. The average of F_z can be obtaining by using the solid angle for a hemisphere, resulting in:

$$\langle F_z \rangle = \frac{F_0}{2\pi} \int_0^{\pi/2} \int_0^{2\pi} \cos \theta d\phi \sin \theta d\theta = \frac{F_0}{2}. \quad (5)$$

The Lambert Law of Scattering states that only $\frac{2}{3}$ of the total thermal radiation emitted from a point will be directed perpendicular to the point of that surface. Applying this coefficient will give a more realistic value of the force of the Yarkovsky force with Eqs. (3) and (5), which result in:

$$F_{rad} = \frac{2\pi\sigma}{3c} R^2 T^4, \quad (6)$$

where R is the radius of the particle. To determine the Yarkovsky force the difference of F_{rad} between the two hemisphere we can either calculate it as the following:

$$F_Y = \Delta F_{rad} = \frac{dF_{rad}}{dT} \Delta T = \frac{8\pi\sigma R^2 T^3}{3c} \Delta T, \quad (7)$$

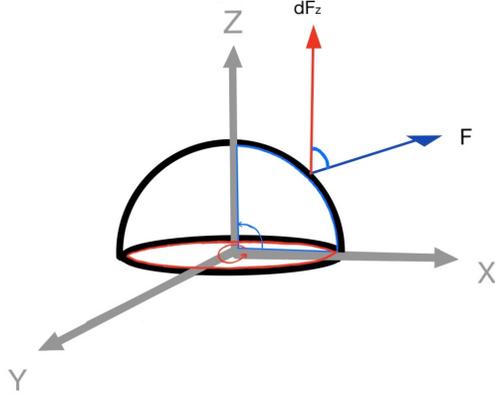


Figure 1: Thermal radiation emitted at all surface area of the semisphere.

or, instead of Eq. (7), we assume that there is no transfer of heat across the two hemispheres hence only one side will have a force acting upon it, resulting in a more simple Yarkovsky force:

$$F_Y = F_{rad} - 0 = \frac{2\pi\sigma}{3c}R^2T^4. \quad (8)$$

3 Results

Eq. (8) represents the Yarkovsky force with no transfer of heat. Spherical objects of various radii were applied to this formula to derive the gravitational force of the sun on these objects, and the Yarkovsky effect upon them. In order to do this, the temperature of a spherical object at 1 AU distance away from the sun needs to be determined using the intensity of the sun. Intensity is defined as the power spread across an area, which will depend on the distance r from the source of power. This can be calculated due to the power from the sun at 1 AU ($1.496 \times 10^{11}\text{m}$) is known as $3.828 \times 10^{26}\text{W}$, and was calculated as follows:

$$I_{\odot} = \frac{P_{sun}}{4\pi r^2} = \frac{3.828 \times 10^{26}\text{W}}{4\pi(1.496 \times 10^{11}\text{m})^2} = 1361.13 \frac{\text{W}}{\text{m}^2}. \quad (9)$$

Table 1: The Gravitational force on an object at 1AU distance from the Sun and Yarkovsky force

Radius (m)	Mass (kg)	Gravitational Force (N)	Yarkovsky Force (N)
10^{-6}	1.257×10^{-14}	7.493×10^{-17}	5.95×10^{-19}
10^{-3}	1.257×10^{-5}	7.493×10^{-8}	5.95×10^{-13}
1	1.257×10^4	7.493×10^1	5.95×10^{-7}
10^3	1.257×10^{13}	7.493×10^{10}	5.95×10^{-1}
10^6	1.257×10^{22}	7.493×10^{19}	5.95×10^5

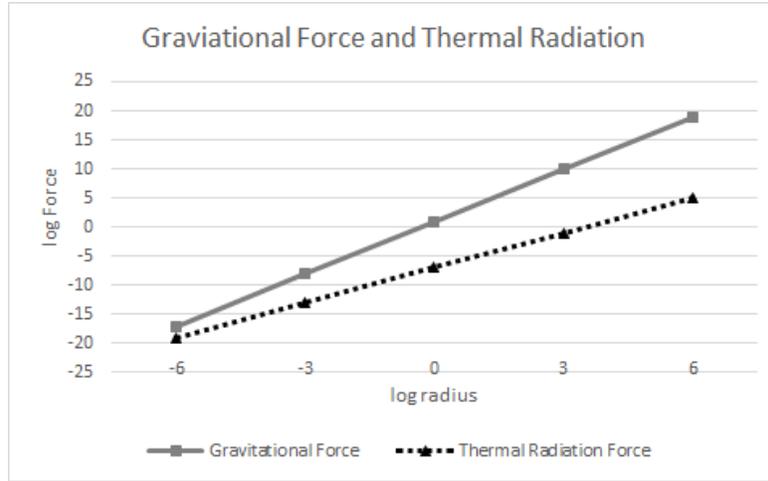


Figure 2: Graphical representation of relationships between radius, gravitational force from the sun, and the force of the Thermal Radiation emitted

Combining Eqs. (4) and (9), the temperature of a spherical object exposed to the sun at 1 AU can be found as:

$$T = \left(\frac{I_{\odot}}{4\sigma} \right)^{\frac{1}{4}}, \quad (10)$$

which is determined to be 278.33K.

4 Conclusion

The Yarkovsky force has been derived for the hypothetical situation where there is no transfer of heat from the side exposed to the sun, to the non-

exposed side. The gravitational and the Yarkovsky force was calculated for various sphere sizes located at 1 AU distance away from the sun and compared, which displayed proportional relationship between the radius and both forces, more specifically $F_G \sim R^3$ and $F_Y \sim R^2$. The larger the radius, the stronger the gravitational force is, and, to a lesser extent, the seasonal effect. The smaller the radius, the Yarkovsky effect becomes more prominent. This does not coincide with how the Yarkovsky effect usually manifests in medium-sized objects, and not very small objects in space. However, this can be explained due to this research not taking into consideration the transfer of heat from the side exposed to the heat source, to the non exposed side. In small objects, this transfer of heat will be rapid, causing an even emission of thermal radiation, making the Yarkovsky force non-existent.

Further research and calculations are required to completely formulate the complete force of the Yarkovsky effect. The transfer of heat within the object will cause a small amount of Yarkovsky effect to be emitted from the side not exposed to the heat source, which this research did not take into account. This effect will be more prominent on smaller objects due to stronger force to the opposite, making the overall net force smaller. Also, the YORP effect was not taken into consideration, which will have a significantly stronger impact on a rotating object.

References

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- Daniel T. Britt, Br. Guy Colsolmagno, L. L. (2007). *Encyclopedia of the Solar System*. Elsevier, San Diego, CA.