1. The following grouped frequency table shows the annual amount of snowfall (in inches) in NYC for the past 135 winter seasons, starting from winter 1869-70.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-9.9</td>
<td>9</td>
</tr>
<tr>
<td>10-19.9</td>
<td>38</td>
</tr>
<tr>
<td>20-29.9</td>
<td>41</td>
</tr>
<tr>
<td>30-39.9</td>
<td>19</td>
</tr>
<tr>
<td>40-49.9</td>
<td>15</td>
</tr>
<tr>
<td>50-59.9</td>
<td>10</td>
</tr>
<tr>
<td>60-69.9</td>
<td>2</td>
</tr>
<tr>
<td>70-79.9</td>
<td>1</td>
</tr>
</tbody>
</table>

For this scenario, identify the following:
- a. Variable
- b. Individual (Subject)
- c. Population
- d. Population size
- e. Shape of the distribution
- f. Is this variable discrete or continuous?
- g. Make a histogram or a bar chart (as appropriate) for this data.

2. A researcher visited 29 randomly selected Starbucks locations and recorded the number of cappuccinos sold at each coffee shop on March 22. He summarized the data in the following frequency distribution table:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-9</td>
<td>2</td>
</tr>
<tr>
<td>10-14</td>
<td>5</td>
</tr>
<tr>
<td>15-19</td>
<td>8</td>
</tr>
<tr>
<td>20-24</td>
<td>10</td>
</tr>
<tr>
<td>25-29</td>
<td>4</td>
</tr>
</tbody>
</table>

For this scenario, identify the following:
- 1. Variable
- 2. Individual (Subject)
- 3. Sample
- 4. Sample size
- 5. Class (bin) width
- 6. Is the variable discrete or continuous?
- 7. How many Starbucks locations sold at least 10 cappuccinos?
- 8. What percent of Starbucks locations sold at least 10 cappuccinos?

3. This table presents the price distribution of shoe styles offered by an online outlet.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-59.99</td>
<td>256</td>
</tr>
<tr>
<td>60-109.99</td>
<td>124</td>
</tr>
<tr>
<td>110-159.99</td>
<td>37</td>
</tr>
<tr>
<td>160-209.99</td>
<td>13</td>
</tr>
<tr>
<td>210-259.99</td>
<td>6</td>
</tr>
<tr>
<td>260-309.99</td>
<td>3</td>
</tr>
<tr>
<td>310-359.99</td>
<td>3</td>
</tr>
<tr>
<td>360-409.99</td>
<td>1</td>
</tr>
</tbody>
</table>

For this scenario, identify the following:
- a. Individual (Subject)
- b. Population
- c. Population size
- d. Variable
- e. Shape of the distribution
- f. Is the variable discrete or continuous?
MAT 119/120 DEPARTMENTAL FINAL EXAMINATION - REVIEW

4. The following frequency table shows the test score distribution for a random sample of 25 students taking an introductory statistics class at a certain college.

<table>
<thead>
<tr>
<th>Score</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 – 40</td>
<td>0.08</td>
</tr>
<tr>
<td>41 – 51</td>
<td>0.04</td>
</tr>
<tr>
<td>52 – 62</td>
<td></td>
</tr>
<tr>
<td>63 – 73</td>
<td>0.24</td>
</tr>
<tr>
<td>74 – 84</td>
<td>0.20</td>
</tr>
<tr>
<td>85 – 95</td>
<td>0.32</td>
</tr>
</tbody>
</table>

a. Find the missing relative frequency.
b. How many students in the sample had a score of at least 63?
c. Total number of students taking an introductory statistics class at this college is 800. Based on the sample data above, estimate the total number of students (in all intro to stats classes) who scored 52-84 on the test.

5. A frequency distribution for the ages of randomly selected 27 students taking a statistics course in a college is given below.

<table>
<thead>
<tr>
<th>Age</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>4</td>
</tr>
<tr>
<td>19</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>21</td>
<td>3</td>
</tr>
<tr>
<td>22</td>
<td>4</td>
</tr>
<tr>
<td>23</td>
<td>2</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>26</td>
<td>1</td>
</tr>
</tbody>
</table>

a. Make a relative frequency histogram for the data. Label axes and units.
b. What is the shape of the distribution?
c. Compute the sample mean
d. Use information from (c) to fill in the blanks in the following statement:
   In the sample of 27 students taking statistics, the average age of a student is about _______.
e. Do you agree with the following statement?
   ‘‘Based on the sample data, we can infer that the average age of students taking a statistics course in a college is no greater than 21.’’
   Choose the best answer below:
   □ Yes, because the data came from a representative sample.
   □ No, because the sample is not representative of the population.
   □ No, because sample mean is different from the population mean.

6. Consider the following data set consisting of test scores of students in a math class:

31, 33, 35, 36, 40, 40, 41, 43, 45, 47, 48, 51, 51, 52, 55, 56, 59, 70, 71, 74, 76, 78, 84, 87, 87, 90, 93

a. Obtain the five-number summary (i.e. Min, Q₁, Q₂=median, Q₃, Max) for the data.
b. Construct a box plot for the data
c. For this frequency distribution, which measure of the center is larger, the median or the mean? (You do not have to calculate the mean to answer this question).
d. Circle the correct choice and fill in the blank:
The median/mean better describes the center of this data set because the shape of this frequency distribution is __________.
e. Complete the following sentence:
The test score does not exceed ______ for 75% of students in this class.
f. Complete the following sentence:
The test score does is at least ______ for 75% of students in this class.

**Correlation and Regression**

7. A study of 10 countries showed that smaller the percentage change in wages, smaller is the percentage change in consumer prices.

a. Identify the explanatory variable and state its units
b. Identify the response variable and state its units
c. Which of the following is a possible value for the linear correlation coefficient between the percentage change in wages and the percentage change in consumer prices?
   A) 0.7   B) -0.65   C) -0.85   D) 1.1

8. Researchers wanted to study the relationship between amounts of fat, sugar, and carbohydrates and the amount of calories in a hamburger. They gathered relevant data about 22 “brands” of fast food hamburgers and obtained the following scatter plots:

   ![Calories vs. Sugar](image1)
   ![Calories vs. Carbohydrates](image2)
   ![Calories vs. Fat](image3)

a. About how many calories would you predict for a burger that has 20 grams of fat?
b. About how many calories would you predict for a hamburger that has 40 grams of carbohydrates?
c. Which prediction is likely to be more accurate? Why do you think this?
d. Which nutrient has the weakest impact on calories? Why do you think this?
e. What does the idea of strength of the correlation tell you about whether a nutrient is a good predictor of calories?
f. What is the direction of the fat/calories graph? What does the direction of the line tell you about the association between the amount of fat and the calories in fast food hamburgers?

9. The scatterplot below relates wine consumption (in liters of alcohol from wine per person per year) and death rate from heart disease (in deaths per 100,000 people) for 19 developed countries.
a. Identify an individual (subject) in this study?

b. What does each dot in this scatterplot represent?

c. What type of correlation is shown in this scatterplot? Circle the correct answer:
   \( \text{Linear/Non-linear/No correlation} \)

d. What is the direction of association between variables? Circle the correct answer:
   \( \text{Positive/ Negative/ None} \)

For questions e. and f. use the equation of the Least-Square Regression LSR line is:

\[
\hat{y} = -22.97x + 260.56
\]

e. Circle the correct choice and fill in the blank in the following statement: As wine consumption increases by 1 liter of alcohol per person per year, the predicted death Rate from heart disease increases/decreases by _______ deaths per _________ people.

f. Find the death rate from heart disease (per 100,000 people) predicted by the model for a country where wine consumption amounts to 5 liters of alcohol from wine per person per year?

10. Suppose that for a certain baseball season, winning percentage, \( y \), and on-base percentage, \( x \), are linearly related by the least squares regression equation:

\[
\hat{y} = 2.94x - 0.4875.
\]

For this baseball season, the lowest on-base percentage was 0.310 and the highest was 0.362.

a. Underline the correct choice and fill in the blank in the following statement:
   As the on-base percentage increases by 5 percent, the predicted winning percentage \( \text{increases / decreases by} \)______

b. Would it be a good idea to use this model to predict the winning percentage of a team whose on-base percentage is 0.156? Why or why not?

c. Based on this model, what would you expect the winning percentage to be for a team with on-base percentage 0.350?
Discrete Probability Distribution

11. In a survey, 500 children ages 6-11 in an elementary school were asked whether they read books for fun every day. Their responses (yes/no), broken down by gender, are summarized in the table.

<table>
<thead>
<tr>
<th></th>
<th>Boy</th>
<th>Girl</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>48</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>182</td>
<td>194</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. What proportion of children in the survey are boys?
b. What proportion of boys in the survey read books for fun every day?
c. What proportion of girls in the survey read books for fun every day?
d. Do the results of the survey suggest that gender of a child has an effect on reading books for fun? Explain

e. Suppose a child is selected at random from this survey. Are events “a child reads books for fun every day” and “a child is a boy” independent? Explain

12. A bin contains 3 red and 4 green balls. 2 balls are chosen at random, without replacement. Let the random variable X be the number of green balls chosen.

a. Explain why X is not a binomial random variable.
b. Use a tree diagram or a sample space to construct a probability distribution table for X.

13. A bin contains 3 red and 4 green balls. 3 balls are chosen at random, with replacement. Let the random variable X be the number of green balls chosen.

a. Explain why X is a binomial random variable.
b. Construct a probability distribution table for X.
c. Find the mean (expected value) of X.
d. Use the law of Large Numbers to interpret the meaning of the expected value of X in the context of this problem.

14. Suppose you play a die-rolling game in which a fair 6-sided die is rolled once. If the outcome of the roll (the number of dots on the side facing upward) is odd, you win as many dollars as the number you have rolled. Otherwise, you lose as many dollars as the number you have rolled. Let X be the profit of the game or the amount of money won or lost per roll. Negative profit corresponds to lost money.

a. What is your profit if the outcome of the roll is 3?
b. Fill out the following probability distribution table
c. Compute the expected value (the mean) of X

d. Explain the meaning of the expected value of X in the context of this problem

e. If you played this game 100 times, how much would you expect to win or lose?

15. Suppose you pay $2 to play a game of chance, in which you toss a coin and roll a die. You are paid $10 if your coin shows a tail and you roll at least a five on the die. Let the random variable X be the profit of the game or the amount of money won or lost per roll. Negative profit corresponds to lost money.

Fill out the following probability distribution table

<table>
<thead>
<tr>
<th>Event</th>
<th>X</th>
<th>P(X)</th>
</tr>
</thead>
</table>

a. Over the long term, what is your *expected* profit (or loss) per game?
b. If you played this game 100 times, how much would you expect to win /lose?

16. Shipments of television set that arrive at a factory have varying levels of quality. In order to decide whether to accept a particular shipment, inspectors randomly select a sample of 10 television sets and test them; if no more than one television set in the sample is defective, the shipment is accepted. Suppose a very large shipment arrives in which 2% of the television sets are defective. Let $X$ be a random variable representing the number of defective television set in the random sample of 10.

a. Explain why $X$ may be treated as a binomial random variable:
   - Identify $n$ (the number of trials):
   - Specify (in words) which event would be defined as a “success”:
   - Explain why the trials may be considered independent:
   - Give the value of $p$ (the probability of a success):

a. What is the probability that this shipment is accepted? (Use a table or the formula).
b. What is the *expected value* of the number of defective television set in the sample?
c. Fill in the blanks in the following sentence:
According to the Law of Large Numbers, if we have obtained many different simple random samples of size _____ from this shipment, the average number of defective television set per sample would be approximately _______

17. A test consists of 10 true/false questions. To pass the test a student must answer at least 8 questions correctly.
   a. If a student guesses on each question, what is the probability that the student will pass the test?
   b. Find the mean and standard deviation of the number of correct answers.
   c. Is it unusual for a student to pass by guessing? Explain.

18. In a group of 40 people, 35% have never been abroad. Two people are selected at random without replacement and are asked about their past travel experience.
   a. Is this a binomial experiment? Why or why not?
   b. What is the probability that in a random sample of 2, no one has been abroad? What is the probability that in a random sample of 2, at least one has been abroad?

   Normal Distribution, Sampling Distribution of the Sample Mean

19. Height (in inches) of basketball players has a bell-shaped distribution with mean 72.5 inches and standard deviation 3.25 inches. A height of 79 inches is at what percentile?

20. Assume that the finishing times in a New York City 10-kilometer road race are normally distributed with a mean of 61 minutes and a standard deviation of 9 minutes. Let X be a randomly selected finishing time. Find
   a. $P(X > 72)$
   b. $P(52 < X < 70)$
   c. Find $P_{95}$ (the 95 percentile point)

21. Assume that the weights of quarters are normally distributed with a mean of 5.67 g and a standard deviation 0.070 g.
   a. If a vending machine will only accept coins weighing between 5.48 g and 5.82 g, what percentage of legal quarters will be rejected?
   b. If the quarters having weights in the lowest 0.5% and highest 0.5% range are to be rejected, what are the two cutoff weights?

22. To qualify for security officers training, recruits are tested for stress tolerance. The scores are normally distributed with a mean of 62 and a standard deviation of 8. If only the top 15 % of recruits are selected, find the cut-off score. What is the percentile rank of that score?

23. If a one-person household spends an average of $62 per week on groceries, find the maximum and minimum amounts spent per week for the middle 50% of one-person households. Assume the standard deviation is $10 and the variable is normally distributed.

24. A study states that for a particular area, the average income per family is $34,569 and the standard deviation is $8256. What percentage of families in that area earn below $25,000?
25. The volume of soft drink in plastic bottles is a normal random variable with mean 16 ounces and standard deviation 0.6 ounces.
   a. If a bottle is selected at random, find the probability that it contains more than 15.8 ounces of soft drink.
   b. A random sample of 25 bottles is selected from a large quantity of filled bottles. Write down the sampling distribution of sample means. Give the mean and standard deviation of the sampling distribution, and compare the shape of the sampling distribution to the shape of the original distribution of soft drink volumes.
   c. Find the probability that the mean volume of soft drink in the 25 sampled bottles is less than 15.8 ounces.

26. A large survey found that an American family generates an average of 17.6 lb of glass garbage each year. Assume normal distribution with the standard deviation of 3.5 pounds.
   a. What proportion of families generates less than 17 lbs of glass garbage each year?
   b. If a family is selected at random, what is the probability that it generated less than 17 lbs of glass garbage in a year?
   c. If a sample of 35 families is randomly selected, what is the probability that the sample mean is below 17 lb?

Confidence Interval Estimation

27. The mean weight of 10 randomly selected newborn babies at a local hospital is 7.14 lbs and the standard deviation is 0.87 lbs. Assume the weight of newborn babies has approximately normal distribution.
   a. Find the margin of error for the 90% confidence interval for the mean weight of all newborn babies at this hospital.
   b. Use information from part (a) to fill in the blanks in the following sentence:

   __________ of all samples of size _____ have sample means within _______ of the population mean.
   c. Find a 90% confidence interval for the mean weight of all newborn babies at this hospital.
   d. Does the confidence interval, at 90% confidence level, provide sufficient evidence that the mean weight of a newborn at this hospital is above 6.5 lb? Write the appropriate inequality to justify your answer.
   e. If you increase the confidence level (1-\( \alpha \)), will the confidence interval estimate be wider or narrower? Explain.

28. An editor wants to estimate average the number of pages in bestselling novels, so that his estimate falls within 20 pages of the true average. Assuming that the standard deviation is 63 pages, how large a sample of bestselling novels is needed to achieve
   a. 90% confidence?
   b. 95% confidence?
   c. Identify the variable in this context
   d. Identify an individual in this context
29. A study of 40 English composition professors showed that they spent, on average, 12.6 minutes correcting a student's term paper. The standard deviation was 2.6 min.
   a. Find and interpret the 90% confidence interval for the mean grading time of all composition papers.
   b. Does the confidence interval, at 90% confidence level, provide sufficient evidence that the mean time English composition professors spend on grading term papers exceeds 11 minutes?
   Write an appropriate inequality to justify your answer.

30. The data represents a sample of the number of home fires started by candles for the past 7 years in a certain city.
   
   5460  5900  6090  6310  7160  8480  9930
   
   (Check yourself: the sample mean is 7047 and sample standard deviation is 1616)
   a. Identify the variable in this set up
   b. Identify an individual in this set up
   c. Find and interpret the 99% confidence interval for the mean number of home fires started by candles each year, assume normal distribution.
   d. Does the confidence interval, at 99% confidence level, provide sufficient evidence that the mean number of home fires started by candles in this city is greater than 5000 per year
   Write an appropriate inequality to justify your answer.

   **Testing of Hypothesis**

31. At the Foremost State Bank the average savings account balance in 2012 was $1300. A random sample of 45 savings account balances for 2013 yielded a mean of $1350 with a standard deviation of $80.
    At the $\alpha = 0.1$ significance level can we conclude that the mean savings account balance in 2013 is different from the mean savings account balance in 2012?

32. The mean retail price for bananas in 1994 was 46.0 cents per pound. Currently, a random sample of 15 markets gave a mean price of 48.4 cents per pound with a standard deviation of 3.5 cents.
    a. Identify the variable in this set up
    b. Identify an individual in this set up
    c. Assuming that the retail price of bananas is normally distributed, is there enough evidence at $\alpha = 0.05$ level of significance to conclude that the current mean retail price of bananas has increased since 1994?

33. A researcher claims that the average wind speed in a certain city is 8 miles per hour. In a random sample of 32 days, the average wind speed is 8.2 miles per hour. The standard deviation of the sample is 0.6 miles per hour. At $\alpha = 0.05$ is there enough evidence to reject the claim?
34. Report by the Gallup Poll stated that on average a woman visits her physician 5.8 times a year. In a random sample of 20 women the number of doctor visits is obtained as follows:

3 2 1 3 7 2 9 4 6 6 8 0 5 6 4 2 1 3 4 1

(Check yourself: the sample mean is 3.85 and the standard deviation is about 2.52)

At $\alpha = 0.01$ can it be concluded that the actual average number of doctor visits is less than 5.8?

---

**Answers**

**Fundamental Concepts, Organizing and Summarizing Data**

1. 
   a. Annual amount of snowfall (in inches)
   b. Any winter season in NYC since 1869
   c. Past 135 winter seasons in NYC
   d. 135
   e. Skewed right
   f. Continuous
   g. Histogram

2. 
   a. Number of cappuccinos sold on March 22
   b. One of the visited Starbucks locations
   c. 29 randomly selected Starbucks locations
   d. 29
   e. 15
   f. Discrete

3. 
   a. A shoe style offered by the outlet
   b. All shoe styles offered the outlet
   c. 443
   d. Price
   e. Skewed right
   f. Discrete

4. 
   a. $1-(0.08+0.04+0.24+0.20+0.32) = 1-0.88 = 0.12$

   b. $25*(0.24+0.20+0.32)=25*0.76=19$

   c. $800*(0.12+0.24+0.20) = 800*0.56 = 448$

5a. 

---
6. 
   a. min = 31, Q1 = 41, Q2 = 52, Q3 = 76, max = 93
   b. 
   c. mean
   d. median, skewed right
   e. 76
   f. 41

**Correlation and Regression**

7. 
   a. Change in wages (in %)
   b. Change in consumer prices (in %)
   c. 0.7

8. 
   a. 400 calories
   b. 600 calories
   c. A prediction based on the amount of fat is likely to be more accurate, because points on the Calories vs. Fat scatterplot are less dispersed (follow the straight line more closely) than points on the Calories vs Carbohydrates scatterplot.
   d. Sugar has the weakest impact on calories. Points on Calories vs. Sugar scatterplot are more dispersed (follow the straight line less closely) than points on the other two scatterplots. This indicates the weakest linear association (correlation) between the two variables.
   e. The stronger is the association (positive or negative) between the nutrient and calories, the more accurate is the prediction of calories based on this nutrient.
   f. The direction of the fat/calories is positive. It indicates that there is a positive association between the amount of fat and the calories in fast food hamburgers. i.e as the amount of fat increases (decreases) the amount of calories also increases (decreases)

9. 
   a. A developed country (one of 19 selected for the study.)
   b. Wine consumption and death rate from heart disease for a country.
   c. Linear
   d. Negative (countries with higher levels of wine consumption tend to have lower death rate from heart decease)
   e. **Decreases**, ~ 27 deaths per 100000 people.
f. 145.71 deaths per 100,000 people

10. a. Increases, 14.7%
b. Not a good idea, because this would be an extrapolation and x-value of 0.156 is well outside of the range of values the regression equation was based upon.
c. \( \hat{y} = 2.94(0.350) - 0.4875 = 0.5415 \)

Discrete Probability Distribution, Binomial Distribution

11a. 46%
b. 20.9%
c. 28.1%
d. Yes, the results of the survey suggest that gender of a child, aged 6-11, has an effect on the probability of reading books for fun every day. In this survey, girls were more likely to read for fun than boys (28.1% chance vs. 20.9%). However, to draw the conclusion about population (all children ages 6-11), the claim must be tested using Hypothesis Testing (Chi-square test for independence).
e. The events are not independent. For independent events, \( P(A \text{ and } B) = P(A) \times P(B) \). \( P(\text{“fun” and “boy”}) = 48/500; P(\text{“fun”}) \times P(\text{“boy”}) = 124/500 \times 230/500. P(\text{“fun” and “boy”}) \neq P(\text{“fun”}) \times P(\text{“boy”}). Thus they are not independent.

12 a. X is not binomial RV, because trials are not independent. Probability of choosing a green ball changes from trial to trial, and depends on the outcome of previous trials, because we choose without replacement.
b. 

<table>
<thead>
<tr>
<th>event</th>
<th>X # of green</th>
<th>P(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RR</td>
<td>0</td>
<td>1/7</td>
</tr>
<tr>
<td>RG or GR</td>
<td>1</td>
<td>4/7</td>
</tr>
<tr>
<td>GG</td>
<td>2</td>
<td>2/7</td>
</tr>
</tbody>
</table>

13a. X is a binomial RV, because the experiment has a fixed number of trials (n=3); each trial results in 2 complimentary possibilities: a ball is green (success) and a ball is red (failure); and the probability of success \( p = 4/7 \) for any trial (trials are independent because we choose with replacement).
c. $E(X)=1.7143$.

d. If the described experiment is repeated a lot of times, the average number of green balls per each attempt is about 1.7

14

a. $3.$

b.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Profit (in $)</th>
<th>Probability</th>
<th>XP(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
<td>1/6</td>
<td>-2/6</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>1/6</td>
<td>3/6</td>
</tr>
<tr>
<td>4</td>
<td>-4</td>
<td>1/6</td>
<td>-4/6</td>
</tr>
<tr>
<td>5</td>
<td>-5</td>
<td>1/6</td>
<td>5/6</td>
</tr>
<tr>
<td>6</td>
<td>-6</td>
<td>1/6</td>
<td>6/6</td>
</tr>
</tbody>
</table>

c. $E(X)=XP(X)=\frac{-1}{2}=-0.50$

d. A person playing the game a lot of times, will lose, on average, 50 cents per game.

e. - $50$ (Lose $50$)

15a.

<table>
<thead>
<tr>
<th>Profit ($)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1/6</td>
</tr>
<tr>
<td>-2</td>
<td>5/6</td>
</tr>
</tbody>
</table>

b. -$0.33$ expected loss of 33 cents per game

c. Lose $33$

16.

a. X is a **binomial** random variable:

- There is a fixed number of trials, n=10
- Each trial has 2 outcomes. “Success”: “a TV set is defective”
- The trials are independent, because the population we choose from is very large.
MAT 119/120 DEPARTMENTAL FINAL EXAMINATION - REVIEW

- Probability of success, \( p = 0.02 \)
  
  b. \( P(X \leq 1) = P(X=0) + P(X=1) = 0.8171 + 0.1667 = 0.9838 \)
  
  c. \( E(X) = np = 0.2 \)
  
  d. 10, 0.2

17

a. 0.055

b. mean = 5, std. dev = 1.581

c. the usual \# of correct answers is 2 to 8. So passing by guessing is not unusual.

18

a. This is not a binomial experiment. We select without replacement and the population we choose from is small (N=40). The outcomes of trials are not independent, their probabilities change depending on outcomes of previous trials.

b. In this population 14 people (35\% of 40) have never been abroad. The probability that in a random sample of 2, no one has been abroad is \((14/40)(13/39)\) or about .117

c. \( P(X \geq 1) = 1 - P(0) = 0.883 \)

Normal Distribution, Sampling Distributions of the Sample Mean

19. 97.72\% or approximately 98th percentile

20.

a. 0.1112

b. 0.6826

c. 75.81

21.

a. 1.96\%

b. 5.49 g and 5.85 g

22. 70.32 (85\textsuperscript{th} percentile)

23. $55.30; $68.70

24. 12.3\%

25

a. \( P(x > 15.8) = P(z > -0.33) = 0.6293 \)

b. The sample means are normally distributed with mean = 16, and standard deviation = 0.12

c. \( P(x\bar{} < 15.8) = P(z < -1.67) = 0.0475 \)

26

a. \( P(x<17) = P(z<-0.17) = 0.4325 \)

b. Same as a.
c. \( P(x-\bar{b} < 17) = P(z < -1.01) = 0.1562 \)

Values of z were rounded to the hundredth and z-table was used. Your answers may differ slightly if you are using software to calculate probabilities.

**Confidence Interval Estimation**

27
a. \( T_{\alpha/2} = 1.8331, E=0.504, \)
b. 90%, 10, 0.504 lb

c. (6.636 lbs, 7.644 lbs)
d. Yes, the interval does not contain the value 6.5 and 6.5 < 6.636

e. Wider. The error (E) will be higher.

28. \( n= (z_{\alpha/2} \sigma/E)^2 \)
a. \( z= 1.64, n=27 \)
b. \( z= 1.96, n=39 \) (because we round UP)
c. number of pages

d. a best-selling novel

29.

29. 
a. \( T_{\alpha/2} = 1.6849 \)
\[ E = 1.6849 \times (2.6)/\sqrt{(40)} = 0.69 \]
\( 11.91 < \mu < 13.29 \)

We are 90% confident that the mean grading time of all composition papers is between 11.91 and 13.29 minutes.

b. The confidence interval, at 90% confidence level, provides sufficient evidence that the mean time English composition professors spend on grading term papers exceeds 11 minutes.

30
a. the number of home fires started by candles in a certain city

b. the year

c. \( 4782.6 < \mu < 9311.4 \)

Sample mean = 7047; \( s=1616 \)
\[ T_{\alpha/2} = 3.7074 \]
\[ E = 3.7074 \times (1616)/\sqrt{(7)} = 2264.4 \]

d. The 99% confidence interval does not provide sufficient evidence that the mean number of home fires started by candles in this city is greater than 5000 per year.

4782.6 < 5000 < 9311.4 (5000 lies within the confidence interval)
Testing of Hypothesis

31. **Critical Value** \( T_{\text{critical}} = 1.6802 \) (2 tailed test, \( \alpha/2 = 0.05 \), d.f.=44)
   Test statistic \( T_o = 4.19 \) is in the critical region (i.e. 4.19 > 1.6802).
   Reject \( H_0 \), sufficient evidence to support the conclusion that the mean savings account balance in 2013 is different from the mean savings account balance in 2012.

   OR
   Test statistic \( T_o = 4.19 \), d.f.=44, **P-value**=0.0001 is less than \( \alpha = 0.10 \) (i.e. 0.0001<0.10)
   Reject \( H_0 \), sufficient evidence to support the conclusion that the mean savings account balance in 2013 is different from the mean savings account balance in 2012.

32. \( T_{\text{critical}} = 1.7613 \) (right tailed test, \( \alpha = 0.05 \), d.f.=14)
   Test statistic \( T_o = 2.656 \) is in the critical region (i.e. 2.656 > 1.7613).
   Reject the null hypothesis \( H_0 \). There is enough evidence (at \( \alpha = 0.05 \)) to support \( H_a \), and to conclude that the retail price of bananas has increased since 1994.

   OR
   Test statistic \( T_o = 2.656 \), d.f.=14, **P-value**=0.0094 is less than \( \alpha = 0.05 \) (i.e. 0.0094<0.05)
   Reject the null hypothesis \( H_0 \). There is enough evidence (at \( \alpha = 0.05 \)) to support \( H_a \), and to conclude that the retail price of bananas has increased since 1994.

33. Note: in this problem, the claim is \( H_0 \)
   \( T_{\text{critical}} = 2.0395 \) (two tailed test, \( \alpha/2 = 0.025 \), d.f.=31)
   Test statistic \( T_o = 1.886 \) is NOT in the critical region (i.e. 1.886 < 2.0395), do NOT reject \( H_0 \) (not enough evidence)
   There is not enough evidence to reject the claim.

   OR
   Test statistic \( T_o = 1.886 \), d.f.=31, **P-value**=0.0687 is greater than \( \alpha = 0.05 \) (i.e. 0.0687>0.05),
   do NOT reject \( H_0 \) (not enough evidence)
   There is not enough evidence to reject the claim.

34. \( T_{\text{critical}} = -2.5395 \) (left tailed test, \( \alpha = 0.01 \), d.f.=19)
   The sample mean is 3.85 and the standard deviation is about 2.52
   Test statistic \( T_o = -3.46 \) is in the critical region (i.e. -3.46 < -2.5395), reject \( H_0 \) (enough evidence)
   There is enough evidence (at \( \alpha = 0.01 \)) to support \( H_a \), i.e to conclude that the actual mean number of doctor visits is less than 5.8

   OR
   Test statistic \( T_o = -3.46 \), d.f.=19, **p-value**=0.0013 is less than \( \alpha = 0.01 \) (i.e. 0.0013<0.01),
   reject \( H_0 \) (enough evidence)
   There is enough evidence (at \( \alpha = 0.01 \)) to support \( H_a \), i.e to conclude that the actual mean number of doctor visits is less than 5.8