LAGUARDIA COMMUNITY COLLEGE, C.U.N.Y.
Department of Mathematics, Engineering, and Computer Science
Review Problems for Final Examination
Pre-Calculus, MAT200

---

1) Circles:

   a) Write the standard form of the equation of the circle with radius $r = 3$ and the center $(1, -1)$.

   b) Find the center and the radius of the circle $2(x - 3)^2 + 2y^2 = 8$.

   c) Find the center and the radius of the circle $x^2 + y^2 - 6x + 2y + 9 = 0$.

2) Let $f(x) = \dfrac{3x + 1}{3x - 4}$, $g(x) = \dfrac{x}{3x - 4}$, and $h(x) = \sqrt{2x - 4}$

   a) Find the domain of $h(x)$.

   b) Find the function $\dfrac{f}{g}(x)$ and its domain.

3) Let $f(x) = \dfrac{1}{x - 2}$, $g(x) = \dfrac{4}{x}$, $h(x) = \ln(x - 3)$, $F(x) = e^{x-3}$

   a) Find the domain of $h(x)$ and $F(x)$.

   b) Find the domain of $(f \circ g)$ and $(g \circ f)$.

4) Let $f(x) = 3x + 2$, $g(x) = x^2 + 2x + 1$, and $h(x) = \dfrac{2x + 1}{x - 1}$

   a) Find and simplify $(g \circ f)(x)$, $(f \circ g)(x)$, $(f \circ f)(x)$.

   b) Find f^{-1} and show that the function you found is indeed the inverse of $f(x)$.

   c) $h(x), x \neq 1$ is one-to-one. Find its inverse and check the result.

5) a) Graph the following piecewise-defined function.

$$f(x) = \begin{cases} -1, & \text{if } x < -2; \\ 2 + x, & \text{if } -2 < x < 2; \\ x^2, & \text{if } x \geq 2. \end{cases}$$

   b) State the domain and range of $f(x)$.

6) Let $f(x) = -2x^2 + 1$ and $g(x) = x^3 + 1$.

   a) Find the average rate of change of $f(x)$ from 1 to 3.

   b) Find the average rate of change of $g(x)$ from 0 to 2.

   c) Find the difference quotient of $f(x)$, i.e., find $\dfrac{f(x + h) - f(x)}{h}$. Simplify.

7) Consider the function $f(x) = \dfrac{x}{x-6}$

   a) Is the point $(3, -1)$ on the graph of f?
   b) If $f(x) = 3$, what is x?
   c) Find the x-intercepts and the y-intercepts if any.

8) Determine algebraically whether each function is even, odd, or neither.

   a) $f(x) = x + x^3$
   b) $g(x) = x^8 + x^2$
   c) $h(x) = x + x^{10}$

9) Perform the following transformations of

$$h(x) = \frac{1}{x^2}$$

   (write your resulting equation in every step below):

   a) Shift $h(x)$ up 3 units.
   b) Shift the result of a) left 2 units.
   c) Reflect the result of b) about the x-axis.
   d) Reflect the result of c) about the y-axis.

10) Perform the transformations of Problem 9 applied to the functions $f(x) = e^{2x}$ and $g(x) = \ln x$. Sketch the resulting graphs.

11) For each of the quadratic functions

   a) $y = x^2 + 2x + 1$
   b) $y = -2x^2 + 2x - 3$

   determine whether its graph opens up or down, find the vertex and axis of symmetry, intercepts if any. Determine if the graphs have max or min and find it. Sketch the graphs.

12) Solve the following quadratic equations by different methods. Indicate the method you use. Write your solution as a set.

   a) $x^2 - 16 = 0$
   b) $x^2 - 5x + 6 = 0$
   c) $6x^2 + x - 2 = 0$
   d) $(x - 1)^2 = 4$

13) A ball is thrown vertically upward from the top of a building 96 feet tall with an initial velocity of 80 feet per second. The distance s (in feet) of the ball from the ground after t seconds is $s(t) = 96 + 80t - 16t^2$. After how many seconds does the ball strike the ground?

14) Find the vertical, horizontal, and oblique (slant) asymptotes, if any, of the rational functions $F(x) = \dfrac{5}{x^2-1}$, $G(x) = \dfrac{x}{(x-1)(x+4)}$, and $R(x) = \dfrac{6x^2+x+12}{3x^2-5x-2}$

15) Robert has available 400 yards of fencing and wishes to enclose a rectangular area. Express the area A of the rectangle as a function of the width w of the rectangle. For what value of w is the area largest? What is the maximum area?

16) Solve the following:

   a) $|x-1|+1<3$
   b) $|x|>2$
   c) $|x+3|=3$

17) Evaluate exactly (without calculator):

   a) $\log_6 \dfrac{1}{36} - \log_2 16 =$
   b) $2\ln\sqrt[3]{e} =$
   c) $-5e^{\ln 3} =$

18) If $4^x = 7$, what does 4^{-2x} equal?

19) a) Express $\log_b \dfrac{\sqrt[5]{x}}{y^3 z^2}$ as a sum and/or difference of logarithms. Simplify.

   b) Write as a single logarithm. Simplify.
   $\log_4(x^2-1) - 5\log_4(x+1)$

20) Solve for x:

   a) $2^{x^2} = 8^x$
   b) $3^{1-2x} = 5^x$
   c) $\log_3(2x-4) = 2$
   d) $\log_3(x+12) + \log_3(x+4) = 2$
   e) $\ln x + \ln(x+2) = 4$

21) Find the amount of money that results from \$1000 invested at 5% compounded quarterly after the period of 18 months.

22) The size P of bacteria of a certain insect population at time t, in days, obeys the law of exponential growth $P(t) = 1000e^{0.02t}$

   a) What is the number of bacteria at $t = 0$ days?
   b) When will the number of bacteria double?
   c) What is the population after 100 days?
   d) When will the insect population reach 3000?

23) Strontium 90 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.024t}$, where A_0 is the initial amount present and A is the amount present at time t, in years. Assume that a scientist has a sample of 500 grams of Strontium 90.

   a) What is the decay rate of Strontium 90?

   b) How much Strontium is left after 10 years?

   c) What is the half-life of Strontium 90?

24) Express the amount that results from 100 dollars invested at 5 percent compounded continuously after a period of 2 years? Set up, do not evaluate.

25) Solve the triangle with sides $a = 2$, $c = 4$, and angle $\alpha = 30°$ (α is opposite to a). State the law that you use.

26) a) Let t be a real number and let $P = (\frac{-\sqrt{3}}{2}, \frac{-1}{2})$ be the point on the unit circle that corresponds to t. Find the values of $\sin t$, $\cos t$, and $\tan t$.

   b) Find

$$\arcsin\left(\sin\frac{2\pi}{3}\right).$$

   Explain your steps. Show all the work. Do not use calculator.

   Note: $\arcsin\theta$ is another notation for $\sin^{-1}\theta$.

27) Solve the following equations on the interval $0 \le \theta < 2\pi$:

   a) $2\cos^2\theta + \cos\theta = 0$

   b) $2\cos^2\theta + \sin\theta = 5$

   c) $2\cos\theta + 2\sin\theta = 2$

28) Determine the amplitude and period of $y = 2\cos(-\frac{\pi}{4}x)$ and sketch the graph of this function. Make sure to indicate key points on your graph.

29) Prove the following identity:

$$\sin\theta(\cot\theta + \tan\theta) = \sec\theta.$$

30) A wire 100 feet long is attached to the top of a radio transmission tower, making an angle of 30° with the ground. How high is the tower? How high is the tower if this angle is 35°?

Some conceptual questions:

31) Give examples of relations that are functions; **Not** functions; one-to-one functions; functions, but Not one-to-one. Explain your examples in full sentences. Use different ways to represent relations.

32) How many x-intercepts can the graph of a function have? How many y-intercepts can the graph of a function have? Explain.

33) Are $f(x) = x$ and $g(x) = \sqrt{x^2}$ the same functions? Why? Explain in full sentences.

34) Can a graph of a function be symmetric about the x-axis? Why or why not? Can it be symmetric about the y-axis? Why or why not?

Additional miscellaneous practice questions:

35) Jim places one thousand dollars in a bank account that pays 5.6 % compounded continuously. After one year, will he have enough money to buy a computer system that costs $1060? If another bank will pay Jim 5.9 % compounded monthly, is this a better deal?

36) A culture of bacteria obeys the law of uninhibited growth. If 500 bacteria are present initially and there 800 after 1 hour, how many will be present in the culture after 5 hours? how long is it until there are 20000 bacteria?

37) A ship, offshore from a vertical cliff known to be 100 feet in height, takes a sighting of the top of the cliff. If the angle of elevation if found to be 25°, how far offshore is the ship?

38) Two sensors are spaced 700 feet apart along the approach to a small airport. When an aircraft is nearing the airport, the angle of elevation from the first sensor to the aircraft is 20°, and from the second sensor to the aircraft it is 15°. Determine how high the aircraft is at this time.

39) Find the domain of $f(x) = \ln(x^2 + 4x - 21)$

40) Solve: $3^{2x} - 3^x - 6 = 0$

41) Establish identity: $\dfrac{1 - \sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1 - \sin\theta} = 2\sec\theta$

Answer Key:

1) Circles:

   a) $(x-1)^2 + (y+1)^2 = 9$

   b) $(3, 0)$ – center; $r = 2$ – radius.

   c) $(3, -1)$ – center; $r = 1$ – radius.

2) a) $D_h = \{x \in \mathbb{R} \mid x \geq 2\}$ – set-builder notation or $[2, \infty)$ – interval notation.
   (NOTE: D_h denotes the domain of h. Similar notation is used below.)

   b) $\dfrac{f}{g}(x) = \dfrac{3x+1}{x}$ and $D_{f/g} = \{x \in \mathbb{R} \mid x \neq 0, x \neq \frac{4}{3}\}$

3) a) $D_h = \{x \in \mathbb{R} \mid x > 3\}$ and $D_F = \mathbb{R}$.

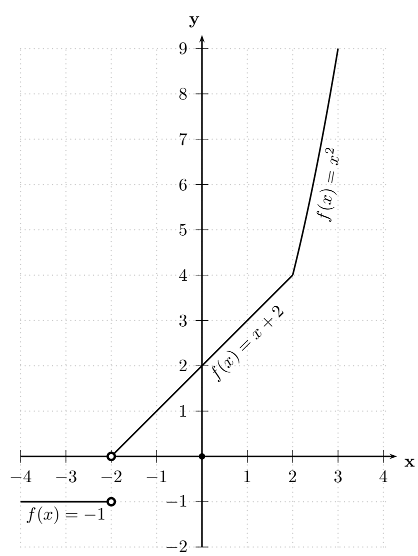
   b) $D_{f \circ g} = \{x \in \mathbb{R} \mid x \neq 0, x \neq 2\}$ and $D_{g \circ f} = \{x \in \mathbb{R} \mid x \neq 2\}$

4) a) $(g \circ f)(x) = 9x^2 + 18x + 9 = 9(x+1)^2$
   $(f \circ g)(x) = 3x^2 + 6x + 5$
   $(f \circ f)(x) = 9x + 8$

   b) $f^{-1}(x) = \dfrac{1}{3}(x-2)$.
   Now check that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.

   c) $f^{-1}(x) = \dfrac{x+1}{x-2}$. Now check that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.

5) a) See the graph below:



   b) $D_f = \{x \in \mathbb{R} \mid x \neq -2\}$,
   Range is $\{-1\} \bigcup (0, \infty)$.

6) a) $\dfrac{f(3) - f(1)}{2} = -8$.

b) $\dfrac{g(2)-g(0)}{2} = 4.$

c) $\dfrac{f(x+h)-f(x)}{h} = -4x - 2h.$

7) a) $f(3) = -1$. Yes.

b) $x = 9$

c) $(0,0)$ is the only $x-$intercept. It is also the $y-$intercept.

8) a) Odd.

b) Even.

c) Neither.

9) a) $y = \dfrac{1}{x^2} + 3$

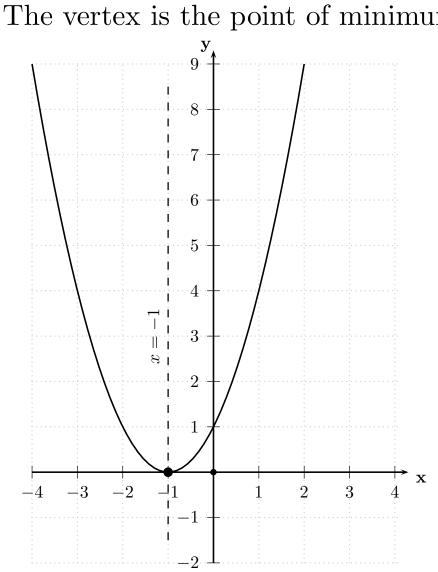
b) $y = \dfrac{1}{(x+2)^2} + 3$

c) $y = -\dfrac{1}{(x+2)^2} - 3$

d) $y = -\dfrac{1}{(2-x)^2} - 3$

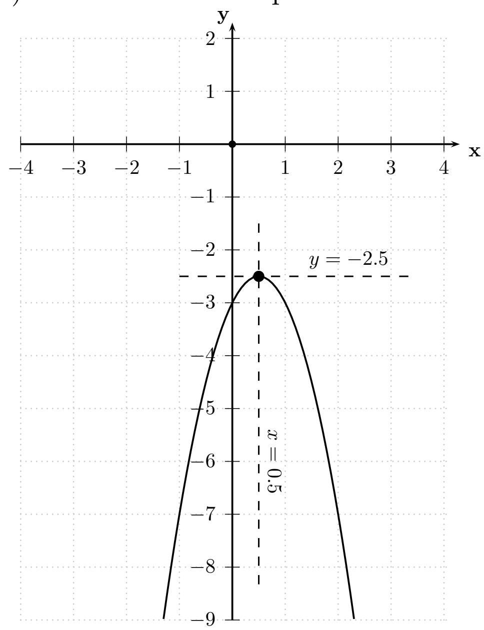
10) Attend the Problem Solving sessions (Pre-Calculus Workshops) offered by the MEC faculty members weekly and work on this exercise.

11) a) Opens Up. Vertex is $(-1,0)$. The axis of symmetry is $x = -1$. The only x-intercept is $(-1,0)$. The only y-intercept is $(0,1)$. The vertex is the point of minimum for this parabola.



b) Opens Down. Vertex is $(\frac{1}{2}, -\frac{5}{2})$. The axis of symmetry is $x = \frac{1}{2}$. There are no x-intercepts.

The only y-intercept is $(0, -3)$. The vertex is the point of maximum for this parabola.



12) a) $\{-4, 4\}$
    b) $\{2, 3\}$
    c) $\{-\frac{2}{3}, \frac{1}{2}\}$
    d) $\{3, -1\}$

13) $t = 6$ seconds.

14) a) For $F(x)$:
    $y = 0$ – horizontal asymptote;
    $x = 1$ and $x = -1$ – vertical asymptotes.
    b) For $G(x)$:
    $y = 0$ – horizontal asymptote;
    $x = 1$ and $x = -4$ – vertical asymptotes.
    c) For $R(x)$:
    $y = 2$ – horizontal asymptote;
    $x = -\frac{1}{3}$ and $x = 2$ – vertical asymptotes.

15) $A(w) = -w^2 + 200w$; $w_{max} = 100\ yd$; $A_{max} = A(w_{max}) = 10,000\ yd^2$

16) a) $(-1, 3)$
    b) $(-\infty, 2) \bigcup (2, \infty)$
    c) $\{0, -6\}$

17) a) -6

b) $\frac{2}{3}$

c) -15

18) $\frac{1}{49}$

19) a) $\frac{1}{5}\log_b x - 3\log_b y - 2\log_b z$

b) $\log_4 \frac{x^2-1}{(x+1)^5} = \log_4 \frac{x-1}{(x+1)^4}$

20) a) $\{0, 3\}$

b) $\frac{\ln 3}{\ln 5 + 2\ln 3}$.
Think how you can simplify it using properties of logarithms.

c) $\{\frac{13}{2}\}$

d) $\{-3\}$. Note that you have to reject $x = -13$. Why?

e) $\{-1 + \sqrt{1+e^4}\}$. Observe rejection of another root again.

21) $A = P\left(1 + \frac{r}{n}\right)^{nt} \approx 1077.38$.

Here $P = 1000$, $r = 0.05$, $n = 4$, $t = \frac{18}{12} = \frac{3}{2}$ (time is in years).

22) a) 1000

b) $\frac{\ln 2}{0.02} \approx 34.66$ days.

c) $P(100) = 1000e^2 \approx 7389$

d) $\frac{\ln 3}{0.02} \approx 54.93$ days.

23) a) 0.02 or 2%

b) $A(10) = 500e^{-0.2} \approx 409.37$ grams.

c) $\frac{\ln 2}{0.02} \approx 34.66$ years.

24) $A = Pe^{rt} = 100e^{0.1}$

25) $\gamma = 90°$, $\beta = 60°$, $b = 2\sqrt{3}$.

The Law of Sines: $\frac{\sin\alpha}{\alpha} = \frac{\sin\beta}{\beta} = \frac{\sin\gamma}{\gamma}$.

26) a) $\sin t = -\frac{1}{2}$, $\cos t = -\frac{\sqrt{3}}{2}$, $\tan t = \frac{\sqrt{3}}{3}$

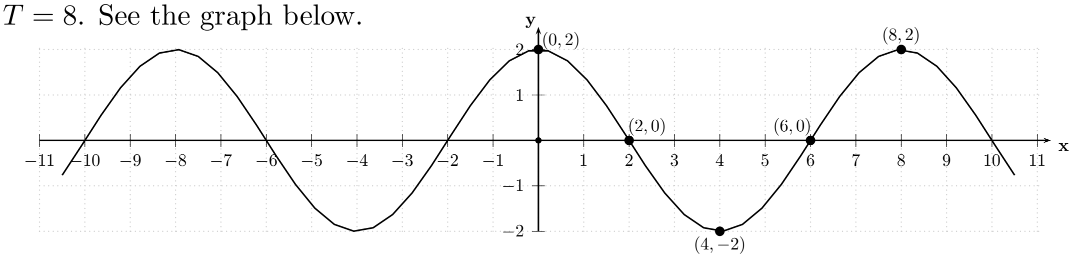
b) $\frac{\pi}{3}$

27) a) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

b) No solutions.

c) $\left\{0, \frac{\pi}{2}\right\}$

28) $A = 2$, $T = 8$. See the graph below.



29) Attend the Problem Solving sessions (Pre-Calculus Workshops) offered by the MEC faculty members weekly and work on this exercise (and other similar exercises).

30) $h = 100 \sin 30° = 50$ feet; $h = 100 \sin 35° \approx 57$ feet.

Conceptual Questions Hints:

31) Think about the graph of a circle $(x - h)^2 + (y - k)^2 = r^2$. Recall the vertical line test. What conclusion can you make? Now think about the graph of a parabola $y = ax^2 + bx + c$. Is it a function? Yes, by the vertical line test. Now recall the horizontal line test. What conclusion can you make? Is it one-to one or not?

This is the line of reasoning you should follow when thinking about this question.

32) Think about the vertical line test.

33) Recall that $\sqrt{x^2} = |x|$. Now answer the question.

34) Once again, think about the vertical line test.

Answers to Additional Miscellaneous Practice Questions:

35) Jim will not have enough money to buy the computer. The second bank offers the better deal.

36) When $t = 5$: there are approximately 5243 bacteria. $t \approx 7.85$ hours.

37) Approximately 214.45 feet

38) Approximately 710.97 feet.

39) $(-\infty, -7) \bigcup (3, \infty)$

40) $\{1\}$

41) Hint: Simplify the left-hand side to get the right-hand side.

Attend the Problem Solving sessions (Pre-Calculus Workshops) offered by the MEC faculty members weekly and discuss these questions (and other similar exercises).